## FLEXURAL ANALYSIS OF LAMINATED PLATE EMBEDDED ON [ELASTIC MEDIUM FOUNDATION](https://link.springer.com/article/10.1007/s00366-020-01107-7) SUBJECTED TO TRANSVERSE LOAD USED IN INDUSTRIES: A RADIAL BASIS FUNCTION APPROACH

Chandan Kumar1, Rahul Kumar2, Appaso M Gadade3, Harish K. Sharma4 and Jeeoot Singh5

1Department of Mechanical Engineering, BIT, Mesra Patna Campus, India

2Department of Mechanical Engineering, Institute of Engineering and Technology, Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur, India

3Thapar Institute of Engineering and Technology, Patiala 147004 India.

4Department of Mechanical Engineering, GLA University, Mathura, India

5Department of Mechanical Engineering, MMMUT, Gorakhpur, India

*\*Corresponding Author: rahul22mech@gmail.com*

***Abstract:*** *The present study developed the radial basis functions based meshless collocation technique (RBFMCT) using higher order shear deformation theory (HSDT) having five unknown variables for the static analysis of elastically supported laminated plates. The governing differential equations (GDEs) for laminated plates embedded on the* [*elastic medium foundation*](https://link.springer.com/article/10.1007/s00366-020-01107-7) *are formulated via Hamilton’s principle. Here, seventeen types of radial basis functions (RBFs) are taken to demonstrate the correctness and consistency of the present solution methodology regarding node number and computational time. In addition, the effects of I, L, and T types of transverse loading, span-to-thickness ratio, aspect ratio, the effect of (RBFs), and the effect of two parameters of elastic foundation on the flexural responses of the laminated plate are also investigated in detail.*

***Keywords****: Flexural analysis, Laminated plate, Elastic foundation, Radial basis function, Transverse loading, Meshfree Technique*.

1. **Introduction**

Plates/panels are one of the important structural elements in aerospace, automotive, marine, and other high-performance engineering structures. During their service life, they are subjected to different loading conditions, and resulting deformations may be moderate to large. These structural components are preferably made up of fiber-reinforced composites stacked in layers or sandwich structures, resulting in saving of weight. There are several plate models that intend to predict the kinematics of these structures more precisely and more effectively. Pagano [1] originated a pioneering work by adducing an exact 3-D elasticity solution for the bending response of laminated during cylindrical loading. In extension to the above work, Pagano [2] presented a 3D elasticity solution for the bending analysis of a rectangular laminate plate. Srinivas and Rao [3] introduced a 3D linear elasticity solution for the structural response of simply supported thick laminated plates with nine elastic constants of orthotropy. Reddy et al. [4] carried out finite element analyses to study the bending response of laminated plates. Savithri and Varadan, [5] investigated an authentic static response of laminated orthotropic plates. Sahoo et al. [6] investigated bending response and natural frequency characteristics of laminated woven glass/epoxy plate via two HSDT models. Karama et al. [7] introduced a new HSDT displacement model for the flexural analysis of laminated plate. Paydar and Libove [8] introduced a finite-difference formulation for a small deflection theory associated with GDEs and a total potential energy formulation for studying flexural elastic sandwich plates. Saood et al. [9] studied the effects of fiber angle on the steady-state response of laminated plates. Khan and Saxena [10] reviewed the mechanical properties of polymer composite under different loading rates. Rodrigues et al. [11] introduced the meshless method to examine the flexural analysis of antisymmetric angle-ply [laminates](https://www.sciencedirect.com/topics/materials-science/composite-laminate) via distinct HSDTs displacement models. Chai et al. [12] examined the bending analysis of laminated columns under uniaxial compression and transverse load via closed-form formulation. Ferreira et al.[13] presented collocation approach with a Deslaurier Dubuc interpolating basis for the flexural and [free vibrations](https://www.sciencedirect.com/topics/engineering/free-vibration) analysis of isotropic and [laminated plates](https://www.sciencedirect.com/topics/engineering/laminated-composite-plate) in the framework of FSDT displacement model. Pavan and Nanjunda Rao [14] used the Isogeometric collocation approach for the linear bending analysis of laminated plates via Reissner–Mindlin theory. Xiao et al. [15] used a meshless technique for the bending study of thick laminated composite elastic plates. Ray [16] investigated the 3D exact solutions for the flexural response of antisymmetric angle ply laminated plates governed by the FSDT displacement model. Shukla et al.[17] studied the natural frequency of eight layered laminated plates using RBF based meshfree method.  Xiang et al.[18] used inverse multiquadric RBF technique for the analysis of isotropic, sandwich, and laminated plates. Xiang et al.[19] used the RBF-based meshfree method for the frequency analysis of the laminated plate. Xiao et al.[20] investigated bending response of laminated plate using meshless local Petrov–Galerkin method with RBF. Dinis et al.[21] used the natural neighbour radial point interpolation method for laminated plate's bending and dynamic analysis. Rodrigues et al.[22] studied the static response of laminates plate using HSDTs and a radial point interpolation method. Belinha et al. [23] used a meshless technique for the static analysis of laminated plates in the framework of the FSDT model. The MQRBF method [24] is the most valuable and has been implemented in various applications. Basically, Hardy proposed the MQRBF for data surface fitting [25], and Kansa used it for calculating the partial differential equations [26]. Ferreira expanded the MQRBF to analyses beams [27], plates [28], and shells [29]. Recently Kumar and Singh [30] implemented MQRBF for the analysis of plates. Tornabene [31] used the RBFs approach for the analysis of laminated shells and panels. Xiang and Kang [32] implemented thin-plate-spline RBF for the analysis of laminated plate. Liew et al. [33] introduced a review on the enhancement of element-free or meshfree methods and their applications for laminated and FGM structures analysis.

In the current work, the flexural analysis of elastically supported laminated plate under the *I, L* and *T* shape of transverse loading is considered under the framework of the HSDT model. From the authors knowledge, the various type of loading used in industries is considered for elastically supported laminated plate, which is not available in the literature. Parametric studies are directed to study the flexural response, and the influence of various influential factors (e.g., elastic foundation, various types of transverse loading, aspect ratio, span-to-thickness ratio, and orthotropy ratio) is evaluated.

1. **Mathematical Formulation**

The laminated plate with length, breadth and thickness are ‘*a*’,’*b*’, and ‘*h*’ which is shown in Figure.1.

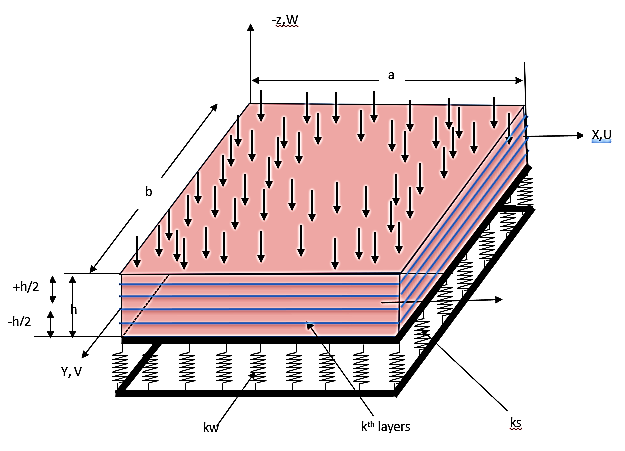


Fig. 1. Laminated plate

The displacement field with the HSDT model is formulated as[34]:



 (1)



where ,  and  are midplane displacements and , are rotations of the normal to the midplane due to shear deformation about *y* and *x*-axes, respectively,  is transverse shear stress function proposed by Kumar et al.[35] which satisfy zero transverse shear stress boundary condition at the top and bottom surfaces of the plate; therefore, a shear correction factor is not required.

The strain displacements relations are formulated as:

 (2)

The stress-strain relations for *kth* layer of the laminated plate is formulated as[36],:

 (3)

The Hamilton’s principle of the laminated plate is written as.

 (4)

where *KE* = Kinetic energy, *UE* = Strain energy, *UEF*= strain energy of the elastic foundation

The *UE* of the elastically supported laminated plate is formulated as [37]

 (5)

The *UEF* of the elastically supported laminated plate is formulated as [38]

 (6)

Where *ks* and *kw* are the equivalent shear and Winkler foundation parameters. ks is used to prevent lateral deformation, while ks is used for transverse deformation.

The potential energy due to transverse loads of the elastically supported laminated plate is formulated as

 (7)

where is transverse pressure.

The *GDEs* of the plate are achieved by cumulating the coefficients of *δu0*, *δv0, δw0, * and  can be formulated as:

 (8)

The axial [force resultants](https://www.sciencedirect.com/topics/engineering/resultant-force),  the bending [moment resultants](https://www.sciencedirect.com/topics/engineering/moment-resultant)  , the additional moment resultants related to the transverse shear function  and the transverse shear force resultants and  used in Eq. (8) are expressed as:

 (9)

The plate stiffness coefficients of the laminated plate are written as

 , *i*, *j* = 1, 2, 6 (10)

  *i, j = 4, 5*  **(**11)

Simply supported (SSSS) boundary condition is considered as:

 (12)

 (13)

1. **Solution Methodology**

The importance of *RBF*-based meshfree methods is that it discretizes the *GDEs* directly and produce a high rate of convergence with good accuracy. In the present analysis, we have considered nodes distribution uniformly for a 2-D rectangular domain having *IN* interior nodes, BN boundary nodes, and *N* is the total nodes which are the sum of *IN* and *BN* which is shown in [39]. Here, we have considered seventeen types of RBFs that are used in various types of computational engineering applications and listed in Table 1. The GDEs with five unknown variables  can be an interpolation in the form of the radial distance between nodes. The radial distance *r* as  for plate where *a* and *b* are the length and breadth of a rectangular plate.

**Table 1.** Different types of RBFs applied in computation applications.

|  |  |  |
| --- | --- | --- |
| Sr. No | RBFs | Shape parameter ‘k’ |
| 1 | Polynomial, g1, |  |
| 2 | Gaussian quadratic, g2, |  |
| 3 | Thin Plate Spline, g3, |  |
| 4 | Wendland’s C2, g4, |  |
| 5 | Wendland’s C4, g5, |  |
| 6 | Wendland C6, g6, |  |
| 7 | Hyperbolic secant, g7, |  |
| 8 | Wu-C2, g8, |  |
| 9 | Wu-C4,g9, |  |
| 10 | Hardy’s Multiquadric, g10, |  |
| 11 | Hardy’s Inverse Quadric, g11, |  |
| 12 | Multi-quadratic, g12, |  |
| 13 | Inverse Multi-quadratic, g13, |  |
| 14 | Generalized Inverse Multiquadratic, g14, |  |
| 15 | Inverse quadratic, g15, |  |
| 16 | Multi-quadratic Shu II, g16, |  |
| 17 | Inverse Multi-quadratics, g17, |  |

where ‘*k*’ is the shape parameter that is responsible for the accurate numerical solution and stability of the method in the computational domain. It is additionally reported that stability and accuracy both simultaneously cannot be ensured.

The variable *u* can be interpolated in the form of the radial distance between nodes. The solution of the *GDEs* (8) is assumed in terms of RBFs for nodes 1: *N*, as;

 (14)

 (15)

 (16)

 (17)

 (18)

where *N* is the total number of nodes.

The *GDEs* are discretized and formulated in compact matrix form as:

 (19)

 (20)

where,

(21)

 (22)

 (23)

 (24)

1. **Results and Discussions**

The flexural response of elastically supported laminated plates under *I*, *L*, and *T*-type transverse loading is investigated in this section. Numerous cases have been examined to show the efficacy and applicability of the present formulation. After the convergence study, 15×15 nodes are used throughout the study. The following material properties are taken throughout the analysis except for variation of *E1/E2*, where *E1* is varied. *E1* =25 *E2*; *G12* = *G13* = 0.5 *E2*; *G23* = 0.2 *E2*; ν12 = 0.25. The normalized deflection and stresses are expressed as:

whereis central deflection and is maximum deflection. *q* for different types of loading conditions is shown in Figure 2.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| **L loading** | **I loading** | **T loading** |

Fig. 2. Various types of loads used in industries.

**4.1. Convergence and validation study**

Table 2 represents the convergence study of normalized central deflection of symmetric cross-ply (0/90/90/0) laminated plate a/b=1, a*/h* = 10, *E*1=25×*E2* subjected to a sinusoidal load. It is observed that the present solution obtained by the seventeen RBFs are converged well and also in good agreement with the result presented in the literature by 3D Quasi solution [40] and 2D HSDT solution by Reddy [41]. It can also be noted that all the RBFs produced good results, and the convergence rate is less than 2% after 13×13 nodes.

**Table 2**. Convergence study of normalized central deflections for laminated plate using several basis functions.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| RBFs | 9×9 | 11×11 | 13×13 | 15×15 | 17×17 | Ref.[40] | Ref.[40] | Ref.[40] | Ref.[41] |
| g1 | 0.6404 | 0.7348 | 0.7167 | 0.721 | 0.717 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g2 | 0.7196 | 0.7196 | 0.7195 | 0.7195 | 0.7195 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g3 | 0.4484 | 0.688 | 0.7139 | 0.7179 | 0.7189 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g4 | 0.6842 | 0.6842 | 0.7167 | 0.721 | 0.717 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g5 | 0.7444 | 0.7321 | 0.7271 | 0.7251 | 0.7202 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g6 | 0.7146 | 0.7162 | 0.7177 | 0.7186 | 0.7192 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g7 | 0.7132 | 0.7212 | 0.7172 | 0.7175 | 0.7187 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g8 | 0.7284 | 0.7264 | 0.7234 | 0.7212 | 0.7204 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g9 | 0.5672 | 0.7059 | 0.7167 | 0.7186 | 0.7192 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g10 | 0.7076 | 0.7129 | 0.7153 | 0.7167 | 0.7175 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g11 | 0.7041 | 0.7107 | 0.7138 | 0.7155 | 0.7165 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g12 | 0.6734 | 0.701 | 0.7107 | 0.7154 | 0.7175 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g13 | 0.6662 | 0.7003 | 0.7181 | 0.7161 | 0.7181 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g14 | 0.6237 | 0.6887 | 0.7044 | 0.7124 | 0.7162 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g15 | 0.6476 | 0.6941 | 0.7078 | 0.7143 | 0.7172 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g16 | 0.7126 | 0.7167 | 0.7185 | 0.7194 | 0.7172 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |
| g17 | 0.708 | 0.7143 | 0.7169 | 0.7181 | 0.7187 | 0.7282 | 0.7161 | 0.7278 | 0.7147 |

Fig. 3 shows the convergence study of for symmetric cross-ply (0/90/90/0) laminated plate subjected to a sinusoidal load considering seventeen RBFs. The plate with a/b=1, *a/h* = 10, *E*1=25×*E2* is taken. It is observed that all the RBFs predict less than 2 % after 13×13 nodes and show good agreement with 3D Quasi solution and 2D HSDT solution. So, based on the convergence study, a 15×15 node is used throughout the study. Figure 4 shows the comparison study of central deflections for a laminated plate for different RBFs and the computational time required. The results show that the RBF g9 requires more time for computation of central deflections followed by RBF g8.



Fig. 3. Convergence study of for laminated plate via seventeen RBFs



Fig. 4. Comparison study of central deflections with the computational speed of several RBFs.

Table 3 Comparison study on normalized central deflections for laminated plate using several basis functions.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| RBFs |  |  |  |  |  |
| Mantari et al.[42] | 0.5100 | 0.542 | 0.306 | 0.023 | 0.323 |
| Tran and Kim [40] | 0.5063 | 0.5371 | 0.3033 | -------- | 0.3262 |
| Tran and Kim [40] | 0.5094 | 0.5410 | 0.3041 | ------- | 0.3248 |
| Tran and Kim [40] | 0.5087 | 0.5416 | 0.3038 | ------- | 0.3226 |
| Karama et al.[7] | 0.509 | 0.541 | 0.306 | 0.023 | 0.316 |
| Reddy and Liu [41] | 0.506 | 0.539 | 0.304 | 0.023 | 0.283 |
| g1 | 0.5081 | 0.5438 | 0.3044 | 0.0212 | 0.2085 |
| g2 | 0.5074 | 0.5403 | 0.305 | 0.0229 | 0.2981 |
| g3 | 0.5064 | 0.5394 | 0.3048 | 0.0203 | 0.2902 |
| g4 | 0.5081 | 0.5438 | 0.3044 | 0.0212 | 0.2085 |
| g5 | 0.5104 | 0.5485 | 0.3129 | 0.0253 | 0.3678 |
| g6 | 0.5069 | 0.5398 | 0.3049 | 0.0217 | 0.2935 |
| g7 | 0.5055 | 0.5389 | 0.3039 | 0.0227 | 0.3942 |
| g8 | 0.5083 | 0.5407 | 0.3052 | 0.0241 | 0.3151 |
| g9 | 0.5069 | 0.5398 | 0.3049 | 0.0217 | 0.2935 |
| g10 | 0.5062 | 0.5403 | 0.3048 | 0.022 | 0.2524 |
| g11 | 0.5063 | 0.5413 | 0.305 | 0.0217 | 0.2183 |
| g12 | 0.5056 | 0.5404 | 0.3046 | 0.0214 | 0.228 |
| g13 | 0.5063 | 0.5408 | 0.3049 | 0.0218 | 0.2354 |
| g14 | 0.5062 | 0.544 | 0.3055 | 0.0207 | 0.1299 |
| g15 | 0.506 | 0.5419 | 0.305 | 0.0213 | 0.1902 |
| g16 | 0.5084 | 0.5419 | 0.3055 | 0.0218 | 0.2471 |
| g17 | 0.5068 | 0.5403 | 0.3049 | 0.0225 | 0.2746 |

**Table 4** Comparison study on normalized deflection of elastically supported (0/90/0/90) laminated plate.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| a/h | Method | Kw=0, Ks=0 | Kw=100, Ks=0 | Kw=100, Ks=10 |
| 5 | Setoodeh and Azizi, [43] | 1.2013 | 0.5457 | 0.2628 |
| Present method (g1) | 1.2006 | 0.5470 | 0.2628 |
| Present method (g10) | 1.2011 | 0.5472 | 0.2627 |
| Present method (g11) | 1.2060 | 0.5481 | 0.2631 |
| 10 | Setoodeh and Azizi, [43] | 0.6802 | 0.4048 | 0.225 |
| Present method (g1) | 0.6854 | 0.4070 | 0.2256 |
| Present method (g10) | 0.6855 | 0.4072 | 0.2257 |
| Present method (g11) | 0.6843 | 0.4067 | 0.2255 |
| 20 | Setoodeh and Azizi, [43] | 0.5500 | 0.3548 | 0.2087 |
| Present method (g1) | 0.5541 | 0.3566 | 0.2093 |
| Present method (g10) | 0.5542 | 0.3569 | 0.1971 |
| Present method (g11) | 0.5505 | 0.3553 | 0.2088 |

Table 3 represents the comparison study on normalized deflection and stresses of symmetric cross-ply (0/90/90/0) laminated plate (a/b=1, a/h=10, E1=25E2) subjected to a sinusoidal transverse load using seventeen RBFs. It is observed that the results obtained for normalized deflection and stresses by all seventeen results are in good agreement with the reported results by Mantari et al.[42], Tran and Kim [40], Karama et al.[7], and Reddy and Liu[41].Table 4 represents the comparison study on a normalized deflection for an antisymmetric cross-ply (0/90/0/90) elastically supported laminated plate (a/b=1, a/h=10, E1=25E2) subjected to a sinusoidal transverse loading. The results obtained by the present method are compared with the reported result in the literature by Setoodeh and Azizi [43]. It is noticed that the present normalized deflection is in good agreement with the analytical solution solved by Setoodeh and Azizi [43] reported in the literature.

* 1. **Numerical examples**

*4.2.1. Influence of the transverse loading.*

The analysis is further extended for a laminated plate subjected to I, L, and T loading. Table 5 shows the influence of *I*, *L,* and *T* type transverse loading on central deflection and stresses of a symmetric cross-ply (0/90/90/0) and an antisymmetric cross-ply (0/90/0/90) laminated plate resting on elastic foundations. The plate with a/b=1, *a/h* = 20, *E*1=30×*E2,* and RBF g10 is considered in the present analysis. It is observed from the results that the *T*-type transverse loading shows more central deflection and normal stresses followed by *I* and *L* type of loading for symmetric cross-ply as well as antisymmetric cross-ply laminates, whereas the trend is different for shear stresses. The effect of the foundation is also investigated for the symmetric cross-ply as well as an antisymmetric cross-ply laminated plate. It is observed that the normalized deflection decreases monotonically along with the increasing *Kw* and *Ks*.

**Table 5.** Influences of transverse loading on normalized deflection and stresses of the elastically supported laminated plate.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (*Kw,Ks*) | | Lamination Type | |  |  |  |  |  |  |  |  |
| (0,0) | (0/90/90/0) | | *L* | | 0.354 | 0.4102 | 0.3899 | 0.1244 | 0.0445 | 1.1107 | 0.1151 |
| *I* | | 0.6756 | 0.6756 | 0.7825 | 0.6611 | 0.0356 | 0.3681 | 0.6488 |
| *T* | | 0.8145 | 0.8145 | 0.9499 | 0.8362 | 0.0446 | 0.4641 | 0.8529 |
| (0/90/0/90) | | *L* | | 0.4035 | 0.4269 | 0.0217 | 0.0065 | 0.0288 | 0.6691 | 0.0655 |
| *I* | | 0.7142 | 0.7142 | 0.0428 | 0.0136 | 0.0389 | 0.253 | 0.9238 |
| *T* | | 0.8576 | 0.8576 | 0.0523 | 0.0165 | 0.0514 | 0.3163 | 1.2565 |
| (10,0) | (0/90/90/0) | | *L* | | 0.3369 | 0.3956 | 0.3688 | 0.1128 | 0.0437 | 1.1039 | 0.1058 |
| *I* | | 0.6476 | 0.6476 | 0.7477 | 0.6405 | 0.0344 | 0.3648 | 0.6322 |
| *T* | | 0.7812 | 0.7812 | 0.9086 | 0.8116 | 0.0432 | 0.4604 | 0.8328 |
| (0/90/0/90) | | *L* | | 0.3834 | 0.4074 | 0.0204 | 0.0061 | 0.0279 | 0.6633 | 0.0559 |
| *I* | | 0.6816 | 0.6816 | 0.0407 | 0.013 | 0.0375 | 0.2458 | 0.9049 |
| *T* | | 0.8189 | 0.8189 | 0.0498 | 0.0157 | 0.0497 | 0.3078 | 1.2331 |
| (0,10) | (0/90/90/0) | | *L* | | 0.1775 | 0.2226 | 0.1781 | 0.0406 | 0.0296 | 0.8051 | 0.0341 |
| *I* | | 0.3623 | 0.3623 | 0.4012 | 0.3946 | 0.0217 | 0.3102 | 0.4351 |
| *T* | | 0.4398 | 0.4398 | 0.4934 | 0.5057 | 0.0277 | 0.4002 | 0.5842 |
| (0/90/0/90) | | *L* | | 0.1957 | 0.2204 | 0.0091 | 0.0027 | 0.0171 | 0.5278 | 0.0004 |
| *I* | | 0.3673 | 0.3673 | 0.0209 | 0.0069 | 0.0227 | 0.1757 | 0.6551 |
| *T* | | 0.4442 | 0.4442 | 0.026 | 0.0084 | 0.0315 | 0.2245 | 0.9001 |
| (10,10) | (0/90/90/0) | | *L* | | 0.1726 | 0.2182 | 0.1721 | 0.0374 | 0.0294 | 0.8024 | 0.0316 |
| *I* | | 0.3541 | 0.3541 | 0.3912 | 0.3885 | 0.0213 | 0.3093 | 0.4301 |
| *T* | | 0.4302 | 0.4302 | 0.4815 | 0.4983 | 0.0273 | 0.3991 | 0.578 |
| (0/90/0/90) | | *L* | | 0.1902 | 0.2156 | 0.0088 | 0.0026 | 0.0168 | 0.5258 | 0.0018 |
| *I* | | 0.3585 | 0.3585 | 0.0203 | 0.0067 | 0.0223 | 0.1738 | 0.6496 |
| *T* | | 0.4337 | 0.4337 | 0.0254 | 0.0082 | 0.031 | 0.2222 | 0.8931 |

|  |  |
| --- | --- |
| Symmetric cross-ply (0/90/90/0) | Antisymmetric cross-ply (0/90/0/90) |
|  |  |
|  |
|  |
|  |
|  |

Fig. 5. Effect of transverse loading on normal and in-plane stresses through the thickness of the laminated plate.

Fig. 5 represents the effect of various types of loads on normal and shear stresses on symmetric cross-ply (0/90/90/0) and antisymmetric cross-ply (0/90/0/90) laminated plate (*a*/*h*=20, RBF = 10, *h*= 1/20, *E1*=30 × *E2*, *Kw*=10, *Ks*=10). It is observed that maximum normal stresses are produced by *T* load followed by *I* load and least by *L* load for symmetric cross-ply and antisymmetric cross-ply laminated plates. It is also observed that the value of and  are zero at the *z*=0 mid-plane. It is observed that the maximum value of  for symmetric cross-ply and antisymmetric cross-ply is generated on the boundary of the plate. It is observed that the get maximum value at *z* = 0 for symmetric cross-ply and antisymmetric cross-ply laminated plates and zero at top and bottom of the plate and similarly is zero at top and bottom of the plate and follows the parabolic shape for all three transverse shear stresses.

*4.2.2 Influence of the span-to-thickness ratio.*

Now, the normalized central deflection for angle ply (45/ -45/ 45/ -45) laminated plate (g10, *E1*=25 × *E2*, a/b=1) subjected to *T* type load is computed for various *a*/*h* using g10 for various foundation parameters as shown in Fig. 6. It is seen that by increasing the value of span to thickness ratio, starts decreasing and by increasing the value of *Kw* and *Ks*.



Fig. 6. Influences of *a*/*h* with the elastic foundation on, for angle ply (45/ -45/ 45/ -45) laminated plate.

*4.2.3 Influence of the ratio E1/E2*

Fig. 7 shows the effect of orthotropic ratio vs elastic foundation for (0/ 45/ 60/ 0) laminated plate (g10, load type= *L*, *a*/*h*=20, *Kw*=10, *Ks*=10, a=b). It is observed that by increasing the values of the orthotropic ratio, the normalized deflection starts decreasing.



Fig. 7. Influences of orthotropic ratio (*E1*/*E2*) on central deflection for the elastically supported laminated plate.

*4.2.4 Influences of elastic foundation*

Fig. 8 represents the influences of elastic foundation on normal and shear stresses for angle ply (0/ 45/ 60/ 0) laminated plate (g10, Load type= *L*, *a*/*h*=20, *E1*/*E2* =20, a/b=1). From Fig. 8, it is observed that by increasing the values of *Kw* and *Ks*, the maximum stresses start decreasing for all the normal and shear stresses. Table 6 represents the 2D contours views on the influence of the elastic foundation of the laminated plate (g10, *a*/*h*=20, *E1*/*E2* =20, a/b=1) on normalized deflection under *L*, *T*, and *I* type transverse loading. It is observed that by increasing the values of *Kw* and *Ks*, normalized deflection starts decreasing.



(*a*)



(*b*)



(*c*)



(*d*)



(*e*)

Fig. 8 Influences of elastic foundation (*Kw*, *Ks*) on normal (*a*) ,(*b*)and shear stresses (*c*) , (*d*) and (*e*) for angle ply (0 /45/ 60/ 0) laminated plate through the thickness

**Table 6.** The 2D contours represent the influence of elastic foundation on normalized central deflection under different types of loading for angle ply (45/ -45/ 45/ -45) laminated plate.

|  |  |  |  |
| --- | --- | --- | --- |
| (*Kw*, *Ks*) | *L*-type | *I*-type | *T*-type |
| (0,0) |  |  |  |
| =0.2783 | =0.5460 | =0.6649 |
| (10,0) |  |  |  |
| =0.2667 | =0.5266 | =0.6418 |
| (0,10) |  |  |  |
| =0.1520 | =0.3167 | =0.3882 |
| (10,10) |  |  |  |
| =0.1481 | =0.3101 | =0.3803 |

1. **Conclusions**

In the present study, elastically supported laminated plate’s flexural analysis was carried out utilizing the meshfree technique. The current displacement model was created using HSDT without the need for a shear correction factor. The governing equations were obtained using the Hamilton principle. Using RBFMCT, strong-formed solutions for the flexural analysis of elastically supported laminated plates with simple support were found. The bending behavior of elastically supported laminated plates has been examined for a variety of factors (Effect of transverse loading, effect of RBFs, span-to-thickness ratio, two variable elastic foundations, orthotropy ratio). The present results were verified and unambiguously demonstrated how quickly the existing simulation approach can determine displacements and stresses.

Based on the current results described in this paper, the following conclusions have been obtained:

* The present solution methodology is capable of finding the stresses and deflection under *I*, *L,* and *T* transverse load.
* All the RBFs are a fast convergence rate with acceptable accuracy.
* The computational speed of RBFs g2, g10, g11, g12, g13, g14, g15, g16, and g17 is good as compared to other RBFs.
* The maximum deflection and stresses are minimum for the *L* type of loading, followed by *I* and *T* type of loading.

The research results and calculations presented in this paper are particularly remarkable because they advance our understanding of how this structure functions mechanically and help us assess, compute, and create mechanical models.

# Declaration of competing interest

The authors of this research have stated that they did not have any competing financial interests or personal ties that could have influenced the findings presented in their work. They have made it clear that their research was conducted with complete objectivity and without any external biases.

# Data availability

No data was used for the research described in the article.

# Conflict of Interest:

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

[1] Pagano NJ. Exact Solutions for Composite Laminates in Cylindrical Bending. Journal of Composite Materials, 3, 398–411 (1969)

[2] Pagano NJ. Exact Solutions for Rectangular Bidirectional Composites and Sandwich Plates. Journal of Composite Materials, 4, 20–34 (1970)

[3] Srinivas S, Rao AK. Bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates. International Journal of Solids and Structures,6, 1463–81 (1970)

[4] Reddy BS, Reddy AR, Kumar JS, Reddy KVK. Bending analysis of laminated composite plates using finite element method. International Journal of Engineering, Science and Technology,4,177–90 (2012).

[5] Savithri S, Varadan TK. Accurate bending analysis of laminated orthotropic plates. AIAA Journal,28,1842–4 (1990).

[6] Sahoo SS, Panda SK, Singh VK. Experimental and numerical investigation of static and free vibration responses of woven glass/epoxy laminated composite plate. Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications , 231,463–478 (2017).

[7] Karama M, Afaq KS, Mistou S. A new theory for laminated composite plates. Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications,223,53–62 (2009)

[8] Paydar N, Libove C. Bending of Sandwich Plates of Variable Thickness. Journal of Applied Mechanics, 55, 419–24 (1988).

[9] Saood A, Khan AH, Equbal MI, Saxena KK, Prakash C, Vatin NI, et al. Influence of Fiber Angle on Steady-State Response of Laminated Composite Rectangular Plates. Materials,15, 5559 (2022).

[10] Khan A, Saxena KK. A review on enhancement of mechanical properties of fiber reinforcement polymer composite under different loading rates. Materials Today: Proceedings, 56, 2316–2322 (2022).

[11] Rodrigues DES, Belinha J, Dinis LMJS, Natal Jorge RM. A meshless study of antisymmetric angle-ply laminates using high-order shear deformation theories. Composite Structures,255,112795 (2021).

[12] Chai GB, Yap CW, Lim TM. Bending and buckling of a generally laminated composite beam-column. Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications,224,1–7(2010).

[13] Ferreira AJM, Castro LMS, Bertoluzza S. A high order collocation method for the static and vibration analysis of composite plates using a first-order theory. Composite Structures,89,424–432 (2009).

[14] Pavan GS, Nanjunda Rao KS. Bending analysis of laminated composite plates using isogeometric collocation method. Composite Structures,176,715–728 (2017).

[15] Xiao JR, Gilhooley DF, Batra RC, Gillespie JW, McCarthy MA. Analysis of thick composite laminates using a higher-order shear and normal deformable plate theory (HOSNDPT) and a meshless method. Composites Part B: Engineering, 39,414–427 (2008).

[16] Ray MC. Three-dimensional exact elasticity solutions for antisymmetric angle-ply laminated composite plates. Int J Mech Mater Des, 17,767–782 (2021).

[17] Shukla V, Vishwakarma PC, Singh J, SIngh J. Vibration Analysis of angle-ply Laminated Plates with RBF based Meshless Approach. Materials Today: Proceedings, 18, 4605–4612 (2019).

[18] Xiang S, Wang K, Ai Y, Sha Y, Shi H. Analysis of isotropic, sandwich and laminated plates by a meshless method and various shear deformation theories. Composite Structures,91,31–37 (2009).

[19] Xiang S, Li G, Zhang W, Yang M. A meshless local radial point collocation method for free vibration analysis of laminated composite plates. Composite Structures, 93,280–6 (2011)

[20] Xiao JR, Gilhooley DF, Batra RC, Gillespie JW, McCarthy MA. Analysis of thick composite laminates using a higher-order shear and normal deformable plate theory (HOSNDPT) and a meshless method. Composites Part B: Engineering, 39, 414–427 (2008).

[21] Dinis LMJS, Jorge RMN, Belinha J. Static and dynamic analysis of laminated plates based on an unconstrained third order theory and using a radial point interpolator meshless method. Computers & Structures, 89,1771–1784 (2011).

[22] Rodrigues DES, Belinha J, Dinis LMJS, Natal Jorge RM. The bending behaviour of antisymmetric cross-ply laminates using high-order shear deformation theories and a Radial Point Interpolation Method. Structures, 32,1589–1603 (2021).

[23] Belinha J, Araújo AL, Ferreira AJM, Dinis LMJS, Natal Jorge RM. The analysis of laminated plates using distinct advanced discretization meshless techniques. Composite Structures,143,165–179 (2016).

[24] Sarra SA. Integrated multiquadric radial basis function approximation methods. Computers & Mathematics with Applications, 51, 1283–1296 (2006).

[25] Hardy RL. Multiquadric equations of topography and other irregular surfaces. Journal of Geophysical Research ,76,1905–1915 (1971).

[26] Kansa EJ. Multiquadrics—A scattered data approximation scheme with applications to computational fluid-dynamics—I surface approximations and partial derivative estimates. Computers & Mathematics with Applications,19,127–145 (1990).

[27] Ferreira AJM, Roque CMC, Martins PALS. Radial basis functions and higher-order shear deformation theories in the analysis of laminated composite beams and plates. Composite Structures,66,287–293 (2004).

[28] Ferreira AJM. A formulation of the multiquadric radial basis function method for the analysis of laminated composite plates. Composite Structures, 59,385–392 (2003).

[29] Ferreira AJM, Carrera E, Cinefra M, Roque CMC. Analysis of laminated doubly-curved shells by a layerwise theory and radial basis functions collocation, accounting for through-the-thickness deformations. Comput Mech,48,13–25 (2011).

[30] Solanki MK, Kumar R, Singh J. Flexure Analysis of Laminated Plates Using Multiquadratic RBF Based Meshfree Method. Int J Comput Methods,15, 1850049, 2017.

[31] Tornabene F, Fantuzzi N, Viola E, Ferreira AJM. Radial basis function method applied to doubly-curved laminated composite shells and panels with a General Higher-order Equivalent Single Layer formulation. Composites Part B: Engineering,55,642–659 (2013).

[32] Xiang S, Kang GW. Meshless Solution of the Problem on the Static Behavior of Thin and Thick Laminated Composite Beams. Mech Compos Mater, 54, 89–98 (2018).

[33] Liew KM, Zhao X, Ferreira AJM. A review of meshless methods for laminated and functionally graded plates and shells. Composite Structures,93, 2031–4201 (2011).

[34] Kumar R, Bajaj M, Singh J, Shukla KK. New HSDT for free vibration analysis of elastically supported porous bidirectional functionally graded sandwich plate using collocation method. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science,236,9109–9123 (2022).

[35] Kumar R, Lal A, Singh BN, Singh J. New transverse shear deformation theory for bending analysis of FGM plate under patch load. Composite Structures, 208, 91–100 (2019). https://doi.org/10.1016/j.compstruct.2018.10.014.

[36] Akavci SS. Analysis of Thick Laminated Composite Plates on an Elastic Foundation with the Use of Various Plate Theories. Mech Compos Mater, 41,445–460 (2005).

[37] Benhenni MA, Adim B, Daouadji TH, Abbès B, Abbès F, Li Y, et al. A Comparison of Closed-Form and Finite-Element Solutions for the Free Vibration of Hybrid Cross-Ply Laminated Plates. Mech Compos Mater,55, 181–194 (2019).

[38] Nedri K, El Meiche N, Tounsi A. Free Vibration Analysis of Laminated Composite Plates Resting on Elastic Foundations by Using a Refined Hyperbolic Shear Deformation Theory. Mech Compos Mater, 49, 629–640 (2014).

[39] Singh J, Shukla KK. Nonlinear flexural analysis of laminated composite plates using RBF based meshless method. Composite Structures,94, 1714–1720 (2012).

[40] Tran LV, Kim S-E. Static and free vibration analyses of multilayered plates by a higher-order shear and normal deformation theory and isogeometric analysis. Thin-Walled Structures,130,622–640 (2018).

[41] Reddy JN, Liu CF. A higher-order shear deformation theory of laminated elastic shells. International Journal of Engineering Science, 23,319–330 (1985).

[42] Mantari JL, Oktem AS, Guedes Soares C. A new higher order shear deformation theory for sandwich and composite laminated plates. Composites Part B: Engineering, 43, 1489–1499. (2012)

[43] Setoodeh AR, Azizi A. Bending and Free Vibration Analyses of Rectangular Laminated Composite Plates Resting on Elastic Foundation Using a Refined Shear Deformation Theory. Iranian Journal of Materials Forming, 2,1–13 (2015).